propagation of only the microstrip mode. Table I gives diode and circuit parameters.

III. PERFORMANCE OF THE VARACTOR-TUNED OSCILLATOR

Fig. 3 shows the output power of a microstrip varactor-tuned oscillator measured as a function of the varactor voltage. With a bias current of 100 mA an output power of 110 ± 15 mW CW is obtained from 58.5 to 60.1 GHz by changing the varactor voltage from 0 to 25 V. Except for the first few volts, the tuning curve is nearly linear. Wider tuning ranges can be achieved by changing the coupling between the two resonators. However, this reduces the output power and gives larger power variations over the tuning range. A compromise between the tuning range and the output power is achieved by adjusting the coupling between the two resonators with a small piece of ceramic ($\epsilon_r \approx 80$). This minimizes the varactor losses giving a relatively narrow tuning range, but large enough for most radio system applications. The output coupling which is less critical is fixed; its magnitude is determined for maximum output power. The corresponding oscillator Q is about 80.

The output power produced by the tunable oscillator is of the same order as that obtained from a fixed frequency 60-GHz microstrip IMPATT oscillator [2], [3] when biased at the same dc current. Thus the modulator circuit shows no significant loss compared to a fixed frequency microstrip oscillator. FM noise is an important parameter in FSK radio system design. Noise measurements show that the rms FM noise of the varactor-tuned oscillator is about the same as that of the fixed-frequency oscillator [2], [3] about 400 $\mathrm{Hz/(kHz)^{1/2}}$. It is sufficiently small not to affect the radio system [4] significantly.

The frequency modulation was measured for different baseband signal shapes. For example, Fig. 4 shows the power spectrum of the output signal when a 50-MHz sinusoidal signal is applied to the varactor. The amplitude of the signal was adjusted to give an index of modulation of about 2.4. Fig. 5 shows a rectangular-pulse-shaped baseband signal which is applied to the varactor for FSK modulation, and the same signal after detection by a FM receiver [3]. Comparison between these two signals shows that the rise time of the rectangular pulses increased by less than 1 ns after being transmitted by the oscillator. This result indicates that the oscillator can sustain a modulation rate of up to 200 Mbit/s.

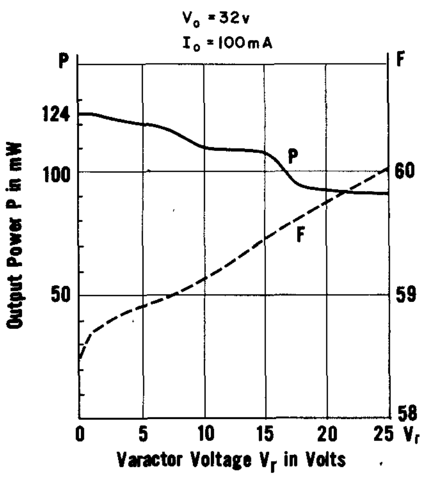

Fig. 3. Output power and frequency of the varactor-tuned microstrip oscillator as a function of the varactor voltage.

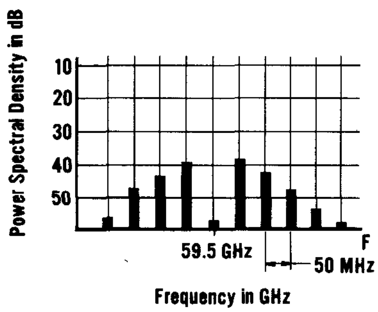

Fig. 4. Power spectrum of the output signal at 59.5 GHz with a sinusoidal signal applied to the varactor to give an index of modulation of 2.4 at a rate of 50 MHz.

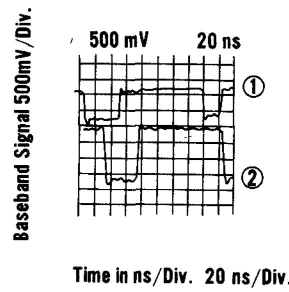

Fig. 5. This figure shows a rectangular pulse-shaped baseband signal applied to the varactor for FSK modulation and the same signal after detection by a FM receiver. The rise time of the input signal is 1.5 ns.

IV. SUMMARY

A hybrid integrated microstrip varactor-tuned oscillator has been built at 60 GHz giving about 100 mW over a 1.6-GHz frequency range. The device can be used as an FM or FSK radio transmitter sustaining modulation rates up to 200 Mbit/s. These results have been obtained with a simple diode package made of two IMPATT diodes, one used as a regular IMPATT oscillating diode, the other one as a varactor for frequency tuning.

ACKNOWLEDGMENT

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Asymmetric Odd-Mode Fringing Capacitances

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Abstract—**An expression is given for the odd-mode fringing capacity of an infinite rectangular bar asymmetrically located inside an infinite U-shaped outer conductor.**

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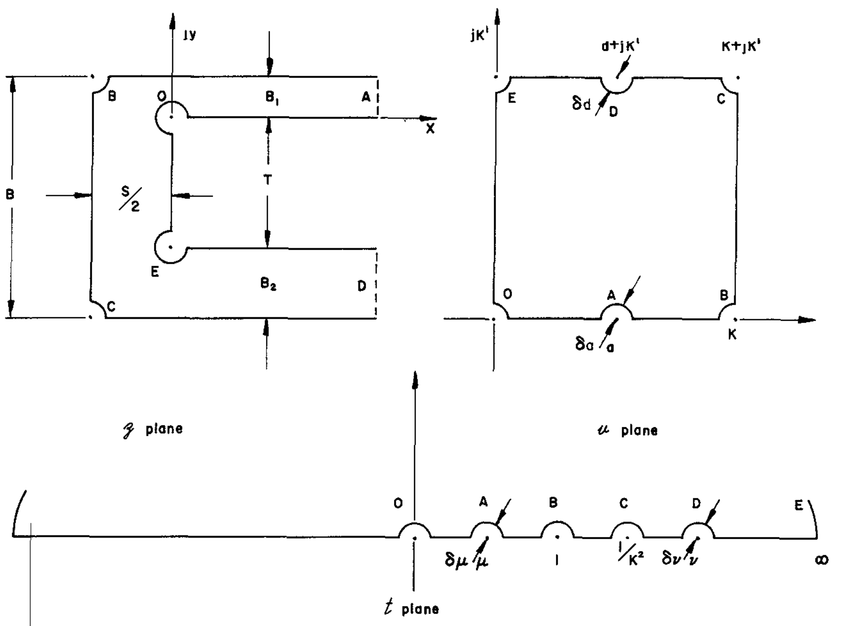

Fig. 1. Coordinate planes of the conformal transformations.

INTRODUCTION

Cockroft [1] and Getsinger [2] have solved, respectively, the problem of determining the odd- and even-mode fringing capacitances for an infinite rectangular bar symmetrically located inside an infinite U-shaped outer conductor by mapping the upper half-plane into an L-shaped infinite polygon by means of a Schwarz–Cristoffel transformation. The essential problem solved in this letter is the evaluation of the integral associated with the Schwarz–Cristoffel transformation which maps the upper half t plane into the doubly infinite U-shaped polygon in the z plane as shown in Fig. 1. This transformation may be written

$$z = \int_0^t \frac{t^{1/2}\,dt}{(\mu - t)\,(\nu - t)\,[(1-t)\,(1-k^2t)]^{1/2}} \tag{1}$$

where

$$0 < \mu < 1 < 1/k^2 < \infty.$$

Points which correspond to each other are labeled similarly in Fig. 1.

The residue R of the integrand at $t = \mu$ is given by

$$R(\mu) = -\mu^{1/2}/(\nu - \mu)\,[(1-\mu)\,(1-k^2\mu)]^{1/2} \tag{2}$$

and this results in a jump at A of $-\pi R(\mu)$, while at $t = \nu$, the residue of integrand $R(\nu)$ is given by

$$R(\nu) = -\nu^{1/2}/(\nu - \mu)\,[(\nu - 1)\,(k^2\nu - 1)]^{1/2} \tag{3}$$

which in turn gives rise to a jump at D of $-\pi R(\nu)$. Both of these jumps are upward of course. Now

$$\frac{1}{(\mu - t)\,(\nu - t)} = \frac{1}{\nu - \mu}\left\{\frac{1}{\mu - t} - \frac{1}{\nu - t}\right\} \tag{4}$$

so that

$$z = \frac{1}{\nu - \mu}\left\{\int_0^t \frac{t^{1/2}dt}{(\mu - t)\,[(1-t)\,(1-k^2t)]^{1/2}} - \int_0^t \frac{t^{1/2}dt}{(\nu - t)\,[(1-t)\,(1-k^2t)]^{1/2}}\right\}. \tag{5}$$

Both of these integrals[1] can be reduced to Jacobi's normal form for elliptic integrals of the third kind by means of the substitution, $t = \mathrm{sn}^2\,u$, if μ and ν are both replaced by $1/k^2\,\mathrm{sn}^2\,\alpha$. Writing $F(x) = \mathrm{sn}\,x/\mathrm{cn}\,x\,\mathrm{dn}\,x$, both integrals of (5) have the value $2F(\alpha)\pi(u,\alpha)$ when α is properly specified. In the first integral, to ensure that $0 < \mu < 1$, let $\alpha = a + jK'$. With a real, $\mu = \mathrm{sn}^2\,a$ so that $0 < a < K$ ensures that $0 < \mu < 1$. In the second integral, to ensure that $1/k^2 < \nu < \infty$, let $\alpha = d$. For d real, $\nu = 1/k^2\,\mathrm{sn}^2\,d$ so that $0 < d < K$ ensures that $1/k^2 < \nu < \infty$. Thus (5) can be evaluated as

$$z = \frac{2}{\mu - \nu}\,\{F(a)\Pi(u, a + jK') + F(d)\Pi(u, d)\} \tag{6}$$

where it is to be kept in mind that $\mu = \mathrm{sn}^2\,a$ and $\nu = 1/k^2\,\mathrm{sn}^2\,d$.

The transformation, $t = \mathrm{sn}^2\,u$, maps the rectangle in the u plane (into the upper half of the t plane) so that points marked with the same letters correspond to each other. Thus it is a simple matter to determine the coordinates of points B, C, and E in the z plane in terms of the independent parameters a, d, and k, by giving u in (6) successively the values K, $K + jK'$, and jK'.

[1] They both occur in the theory of symmetrical fringing capacitances.

THE DIMENSIONS

In the interest of brevity and in view of the direct formal nature of the derivations, the results are presented without further discussion. If $Z(x)$ is the Jacobi zeta-function and we put

$$\bar{Z}(x) = Z(x) + 1/F(x)$$

$$Z(B) = \frac{2}{\mu - \nu}\left\{F(a)\left[K\bar{Z}(a) - j\frac{\pi}{2}\right] + F(d)KZ(d)\right\} \tag{7}$$

$$Z(C) = \frac{2}{\mu - \nu}\left\{F(a)\left[(K + jK')\bar{Z}(a) + j\frac{\pi}{2}\left(\frac{a}{K} - 1\right)\right] + F(d)\left[(K + jK')Z(d) + j\frac{\pi}{2}\frac{d}{K}\right]\right\} \tag{8}$$

$$Z(E) = \frac{2j}{\mu - \nu}\left\{F(a)\left[K'\bar{Z}(a) + \frac{\pi}{2}\left(\frac{a}{K} - 1\right)\right] + F(d)\left[K'Z(d) + \frac{\pi}{2}\left(\frac{d}{K} - 1\right)\right]\right\} \tag{9}$$

then the geometrical dimensions, defined in Fig. 1, are given by

$$B_1 = \frac{\pi}{\nu - \mu}\,F(a)$$

$$B_2 = \frac{\pi}{\nu - \mu} F(d)$$

$$B = \frac{2}{\nu - \mu}\left\{F(a)\left[K'\bar{Z}(a) + \frac{\pi a}{2K}\right] + F(d)\left[K'Z(d) + \frac{\pi d}{2K}\right]\right\}$$

$$S = \frac{4K}{\nu - \mu}\{F(a)\bar{Z}(a) + F(d)Z(d)\}$$

$$T = \frac{2}{\nu - \mu}\left\{F(a)\left[K'\bar{Z}(a) + \frac{\pi}{2}\left(\frac{a}{K} - 1\right)\right] + F(d)\left[K'Z(d) + \frac{\pi}{2}\left(\frac{d}{K} - 1\right)\right]\right\}. \tag{10}$$

THE ODD-MODE FRINGING CAPACITANCE

The odd-mode fringing capacitance of the structure is defined as the limiting value of the difference between the total capacitance of the structure, measured to the magnetic walls, A and D, and the parallel-plate capacitance measured between $0A$ and ED, as the magnetic walls tend to infinity. We thus require the total capacitance of the structure in the z plane out to the magnetic walls at A and D. As we have seen, this figure maps into the upper half t plane so that the magnetic walls at A and D transform into semicircles centered about the points μ and ν, respectively. It is convenient to denote the radii of these circles by $\delta\mu$ and $\delta\nu$ as they approach zero. The capacitance of the structure in the t plane in which one conductor is the line segment between $\mu + \delta\mu$ and $\nu - \delta\nu$ and the other is the infinite line segment between $\nu + \delta\nu$ and $\mu - \delta\mu$ is required, subject to the additional condition that the lines of force are constrained so that the semicircular lines about the endpoints of these line segments are magnetic walls. Riblet [3] has shown recently that the limiting value of this capacitance differs from that of the same structure, in which the endpoints of the line segments are joined by magnetic walls which fall on the real axis, by an amount that is expressible in terms of an excess capacitance, $C_{ex} = \log(2)/\pi$. The capacitance of the structure in the t plane which falls entirely on the real axis is given by $K'(k_0)/K(k_0)$, where

$$k_0^2 = (b - a)\ (d - c)/(d - b)\ (c - a)$$

and

$$a = \mu - \delta\mu, \qquad b = \mu + \delta\mu, \qquad c = \nu - \delta\nu$$

and

$$d = \nu + \delta\nu.$$

Thus

$$k_0^2 = 4\delta\mu\delta\nu/(\nu - \mu)^2$$

in the limit as $\delta\mu$ and $\delta\nu \to 0$. Now

$$\frac{K'(k_0)}{K(k_0)} \approx \frac{1}{\pi}\log\frac{16}{k_0^2} = \frac{1}{\pi}\log\frac{4(\nu - \mu)^2}{\delta\mu\delta\nu}. \tag{11}$$

This capacitance exceeds that of the actual structure by $2C_{ex}$ since there are two vanishing semicircles. If C_0 is the total capacitance of the structure in Fig. 1, then

$$C_0 = \frac{K'(k_0)}{K(k_0)} - \frac{2\log(2)}{\pi} = \frac{1}{\pi}\log\frac{(\nu - \mu^2)}{\delta\mu\delta\nu}. \tag{12}$$

The determination of the parallel-plate capacitances, C_{P_A} and C_{P_D} associated with the plate gaps B_1 and B_2, respectively, proceeds in a strainghtforward, purely formal manner from (6) and will not be given here in the interest of brevity. If we call the odd-mode fringing capacitance for the asymmetrical case C_{f0}'', it is given by the limiting value of $C_0 - C_{P_A} - C_{P_D}$. By substituting one finally obtains

$$C_{f0}'' = \frac{2}{\pi}\Bigg\{a\bar{Z}(a) + dZ(d) + \log H'(0) - \tfrac{1}{2}\log(H(2a)H(2d)) + \log[(1 - k^2\,\mathrm{sn}^2\,a\,\mathrm{sn}^2\,d)/2k] - \tfrac{1}{2}\log(\mathrm{sn}\,a\,\mathrm{cn}\,a\,\mathrm{dn}\,a\,\mathrm{sn}\,d\,\mathrm{cn}\,d\,\mathrm{dn}\,d) + \frac{F(d)}{F(a)}aZ(d) + \frac{F(a)}{F(d)}d\bar{Z}(a) + \frac{1}{2}\left(\frac{F(d)}{F(a)} + \frac{F(a)}{F(d)}\right)\log\frac{\theta(a + d)}{\theta(a - d)}\Bigg\} \tag{13}$$

where $H(x)$ and $\theta(x)$ are the familiar Jacobi theta-functions.

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Transmission-Line Transformation Between Arbitrary Impedances

T. A. MILLIGAN

***Abstract*—An analytical method for transforming between two complex impedances using a single transmission-line matching section is described.**

In a recent letter, Day [1] presents a graphical method to find an impedance transformer using a single transmission-line section. This can also be done analytically using the following formula to transform:

$$Z_1 = R_1 + {}_jX_1$$

to

$$Z_2 = R_2 + {}_jX_2.$$

The transforming line impedance is given by

$$Z_L = \left(\frac{R_1 \cdot |Z_2|^2 - R_2 \cdot |Z_1|^2}{R_2 - R_1}\right)^{1/2}.$$

The transforming line length is given by

$$B = \tan^{-1}\left(\frac{Z_L \cdot (R_2 - R_1)}{R_2 \cdot X_1 + R_1 \cdot X_2}\right)$$

in degrees (or radians).

If B is negative, add 180 (Pi) to get proper length. If the transformation is not possible, Z_L^2 will be negative.

The method can be easily applied on a hand calculator or computer and proves to be much faster and more accurate than a graphical technique.

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